

40. a. Find the area of the triangle above.
- b. Suppose the base of the triangle above is increased by 10% and the height is decreased by 10%. What is the ratio of the area of the new triangle to that of the original triangle? Show or explain how you found your answer.
- c. Consider any triangle with a base,  $b$ , and a height,  $h$ . Suppose the base of the triangle is increased by 10% and the height is decreased by 10%. Will the ratio of the area of the new triangle to that of the original triangle remain the same for all values of  $b$  and  $h$ ? Justify your answer mathematically.

## CONSTRUCTED-RESPONSE SCORING GUIDE

Score	Description
4	Student demonstrates a thorough understanding of the area formula for triangles by correctly determining ratios of areas in both a particular and a general case.
3	Student demonstrates a general understanding of the area formula for triangles by determining ratios of areas in both a particular and a general case, with only minor errors and/or omissions.
2	Student demonstrates a basic understanding of the area formula for triangles by correctly completing or using correct strategies to complete a significant portion of the required tasks.
1	Student demonstrates minimal understanding of the area formula for triangles.
0	Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No response.

### Training Notes for Constructed-Response #40

Score	Description
4	5 points
3	4 points OR 3 points with points from all three parts
2	2 or 3 points
1	1 point OR Student shows minimal understanding of area.

#### Part a (1 point)

- 1 point for correct area [40 (cm<sup>2</sup>)]

#### Part b (2 points)

- 2 points for correct ratio (100:99 or equivalent) with work shown and/or justification
- OR
- 1 point for correct answer with no work shown or justification **or** for correct strategy with arithmetic, or ratio conversion error

**Part c (2 points)**

- 2 points for correct ratio (100:99 or equivalent) with work shown and/or justification  
OR
- 1 point for correct answer with no work shown or justification **or** for correct strategy with arithmetic or ratio conversion error

**Solution Notes**

**Part a:** area of original triangle:  $\frac{1}{2} \times 8 \times 10 = 40$  (cm<sup>2</sup>)

**Part b:** area of new triangle:  $\frac{1}{2} \times 7.2 \times 11 = 39.6$  (cm<sup>2</sup>)  
ratio: 40:39.6 or 1:0.99 or 100:99

**Part c:** area of original triangle:  $\frac{1}{2}bh$   
area of new triangle:  $\frac{1}{2}(1.1b \cdot 0.9h) = \frac{1}{2}(0.99bh)$   
ratio is  $\frac{1}{2}(bh) : \frac{1}{2}(0.99bh)$  or 1:0.99 or 100:99 or equivalent

- Student is not required to explicitly state areas. A student may notice that there is no need to deal with the ( $\frac{1}{2}$ )s and just drop them. However, if a student explicitly identifies an area as base times height, do not rate response as a 4; otherwise, do not penalize.
- Student may read entire problem, do part c, and then use it for part b. As long as student refers to the general case in his/her work on part b, he/she is eligible for a 4-rating.
- If student uses specific cases to “justify” answer to part c, at least two examples must explicitly demonstrate equivalent ratios to receive credit. Answer to part b may constitute one example.
- Student may reverse ratios throughout, as long as they are consistently in the same order, without penalty.



40.

3

$$A: A = \frac{1}{2}bh = 5 \cdot 8 = 40 \text{ cm}^2$$

$$B: A = \frac{1}{2}(1.1b)(.9h) = .495bh, \text{ ratio} = 99:100 \text{ or } \frac{99}{100}$$

C: yes, the Area formula ( $A = \frac{1}{2}bh$ ) is constant for all triangles

$$\frac{\frac{1}{2}bh}{\frac{1}{2}(1.1b)(.9h)} = \frac{.5bh}{.495bh} = \frac{.5}{.495} = \frac{100}{99}, \text{ where } \frac{100}{99} \text{ is a constant (just like pi)}$$

40.

3

A.

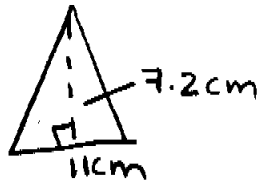


$$\frac{1}{2}bh \quad \text{Area} = 40 \text{ cm}^2$$

$$\frac{1}{2}(b)(8)$$

B  $b = 10 + 10\%(10) = 10 + 1 = 11$

$h = 8 - 10\%(8) = 8 - .8 = 7.2$



The ratio of the new triangle to that of the original is  $39.6:40 \text{ cm}^2$  a 1% difference

C.  $h + (10(b/100))$   $10 - 10\%(1) = 9$

$b - (10(b/100))$   $= 8.8 = 39.6:40$

the ratio will stay the same for all values of  $b$  and  $h$  because they are all %'s of the original. One increases the other decreases by the same amount making the ratio continue being the same.

40.

$$a) \text{ Area} = \frac{1}{2}bh$$

$$\text{Area} = \frac{1}{2} \cdot 10 \cdot 8$$

$$\text{Area} = 40$$

$$b) \text{ Base} = 10\text{cm} \times .10 = 11$$

$$10\text{cm} + 1 = 11\text{cm}$$

$$\text{Height} = 8\text{cm} \times .10 = .8$$

$$8\text{cm} - .8 = 7.2\text{cm}$$

$$\text{Area} = \frac{1}{2}(11)(7.2)$$

$$\text{Area} = 39.6$$

$$39.6 \times 5 = 198 \div 2 = 99$$

$$40 \times 5 = 200 \div 2 = 100$$

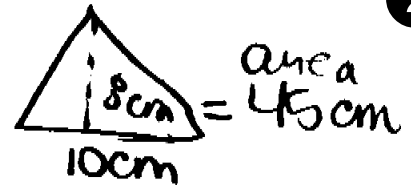
100	99
old	new

c) No because

2

40.

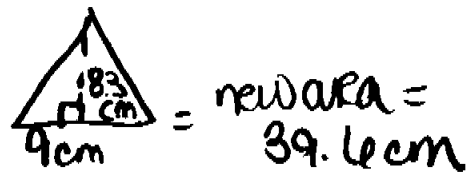
a) Area of the triangle = 45cm



b) new area of triangle

$$39.6 : 45$$

$$.88 : 1$$



c) The ratio is going to be .5:1 because you are decreasing the base and they you cut that in half and multiply by the height to get your area.

2

40.

a  $A = \frac{1}{2}bh$

$A = \frac{1}{2}(10)(8)$

$A = 40$

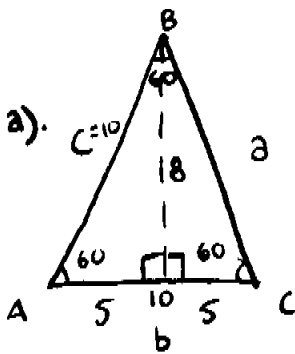
b  $8:10$  or  $4:5$

$7:2:11$

c. no, the area would be off  
 area is  $\frac{1}{2}bh$  and  $b$  and  $h$   
 cause the area would too.

40.

Part a.)



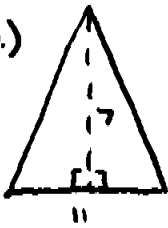
Each angle is  $60^\circ$  ( $180/3 = 60$ )

$$\frac{b}{\sin(B)} = \frac{c}{\sin(C)} \left[ \frac{10}{\sin(60)} = \frac{c}{\sin(60)} \right] \sin 60$$

$A = \frac{1}{2}bh$   $A = \frac{1}{2}(10)(8)$   $c = 10$

$A = 40 \text{ cm}^2$

Part b.)



New base = 11 cm  
 New height = 7 cm

$10 \times .1 = 1$   $10 + 1 = 11$   
 $.8 \times .1 = .7$   $.7 \times 10 = 7$

$A = \frac{1}{2}bh$   $A = \frac{1}{2}(11)(7)$   
 $A = 38.5$

Part c.) No, they will not.

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(15)(5)$

$A = 37.5$

$A = \frac{1}{2}(16)(4)$

$A = 32$

$\frac{32}{37.5} = .853$

The ratio of the two compared is that the  
 New area is 15% less than the original.

<b>Country</b>	<b>1997 Population (in millions)</b>	<b>Estimated 2002 Population (in millions)</b>
China	1221.6	1272.3
India	967.6	1042.3
United States	268.0	279.5
Indonesia	209.8	225.6

41. This table shows the 1997 and estimated 2002 populations of the four most populous countries in the world. Jessica and Katy are using the data for a presentation of their world geography report.
- Jessica says that between 1997 and 2002 the population of China increased faster than that of the United States. Katy insists that the United States population increased more quickly. Explain how **each** of them can be correct. Justify your answer mathematically.
  - Suppose the table was extended for additional 5-year periods (2007, 2012, 2017, etc.). If the population of the United States continues to grow at the percentage rate that is predicted in this table, after how many 5-year periods will the population reach 300 million? Show or explain how you found your answer.

## CONSTRUCTED-RESPONSE SCORING GUIDE

Score	In this item, the student—
4	Student demonstrates a thorough understanding of data analysis by explaining the difference between percent increase and numerical increase and correctly applying percent change to predict future population growth.
3	Student demonstrates a general understanding of data analysis by explaining the difference between percent increase and numerical increase and applying percent change to predict future population growth, with only minor errors or omissions.
2	Student demonstrates some understanding of data analysis by explaining the difference between percent increase and gross increase or applying percent change to predict future population growth.
1	Student demonstrates minimal understanding of percent and/or numerical increase.
0	Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No response.

### Training Notes for Constructed-Response #41

Score	Description
4	4 points
3	3 points
2	2 points
1	1 point OR demonstrates minimal understanding of percent and/or numerical increase

**Part a (2 points)**

2 points for correct explanation: China's increase was greater in terms of absolute numbers (50.7 million vs. 11.5 million) but the United States increase was greater in terms of percentage (4.29% vs. 4.15%) (may also show that U.S. population as percent or ratio of China population increased from 1997 to 2002).

OR

1 point for only one part of explanation correct or for correct strategy in both parts with arithmetic or minor procedural error(s) in each or for correct explanation without supporting arithmetic or percent values.

**Part b (2 points)**

2 points for correct conclusion that United States will reach 300 million in 2 periods following 2002 or 3 periods following 1997 (i.e., 2012), based on 4.29% increase per 5-year period (or percent for United States found in part a).

OR

1 point for correct strategy with error(s) in carrying out computations or interpreting results (e.g., doesn't find minimum number of periods) or for correct answer ("2 5-year periods") without work shown as support.

Note: Do not penalize in either part for rounding, or for incorrect rounding that does not affect the validity of the student's argument.

41. a. Jennifer's statement that China is increasing faster is correct because its total population increased more. Katy's statement that the US population is increasing more quickly is correct because it is growing at larger %.

4

$$\begin{array}{l} \text{China } \uparrow = 1272.3 - 1221.6 = 50.7 \text{ mill} \\ \text{US } \uparrow = 279.5 - 268.0 = 11.5 \text{ mill} \end{array} \left. \vphantom{\begin{array}{l} \text{China } \uparrow \\ \text{US } \uparrow \end{array}} \right\} \text{China is faster}$$

$$\begin{array}{l} \text{China } \% \uparrow = (1272.3/1221.6) - 1 = 4.15\% \\ \text{US } \% \uparrow = (279.5/268) - 1 = 4.29\% \end{array} \left. \vphantom{\begin{array}{l} \text{China } \% \uparrow \\ \text{US } \% \uparrow \end{array}} \right\} \text{US is faster}$$

b.  $2007 = 279.5 \times 1.0429 = 291.49 \text{ mill}$

$2012 = 291.49 \times 1.0429 = 303.996 \text{ mill}$

After 2 more 5 year periods the US population will reach (and pass) 300 million.

a)

$$1272.3 - 1221.6 = 50.7 \quad 50.7 > 11.5$$

$$279.5 - 268 = 11.5$$

$$\text{However.. } \frac{50.7}{1221.6} < \frac{11.5}{268}$$

$$\log_{10} 100 = 2$$

$$10^2 = 100$$

b) After 2 five-year periods

$$A(1+r)^t \quad 300 = 279.5 \left(1 + \frac{11.5}{268}\right)^t$$

$$1.0733... = \left(1 + \frac{11.5}{268}\right)^t$$

$$\log_{\left(1 + \frac{11.5}{268}\right)} \frac{300}{279.5} = t \quad \rightarrow \quad \frac{\log \frac{300}{279.5}}{\log \left(1 + \frac{11.5}{268}\right)} \approx 1.6846$$

41.

3

a)  
China's population increased  
by 50.7 million

The U.S.'s population  
increased 11.5 million

This makes Jennifer right

China's population increased  
by  $3.98\%$   $\left(\frac{50.7}{1272.3}\right)$ .

The U.S.'s population increased  
by  $4.11\%$   $\left(\frac{11.5}{279.5}\right)$

This made Katy right

b)

1997	2002	2007	2012
268 mil.	279.5 mil.	291.0 mil.	303 mil.
*1.0411		*1.0411	

41.

3

a. Jennifer is correct in that the number of millions of people gained total is greatest in China.

$$\begin{array}{r} 1272.3 \\ -1221.6 \\ \hline \end{array}$$

50.7 million people gained.

However, Katy is right in that the U.S. has a higher percent gain than China.

$$\text{China } \frac{50.7}{1221.6} = 0.041 \dots \text{ or about } 4\%$$

$$\text{U.S. } \frac{279.5}{268.0} = \frac{11.5}{268} = 0.0429 \text{ still about } 4\% \text{ but slightly higher than China}$$

$$b. 300 = 268 + 0.0429(268)x$$

$$32 = 0.0429(268)x$$

$$32 = 11.4972x$$

$$x = 2.783 \text{ 5 year periods}$$

$$\begin{array}{r} \sqrt{268 + 0.0429(268)} \\ \downarrow \\ 5 \text{ years } 11.49 \text{ or } 11.5 \text{ million} \\ \downarrow \\ 268 \\ + 11.5 \\ \hline 279.5 + 279.5(0.0429) \\ \downarrow \\ 11.86 \text{ million or } 11.9 \\ 291.4 \text{ million} \\ \text{at } 2012 \end{array}$$

3.5 year periods

one can only reason in 5 more years, 2017, population would exceed 300 million.

41.

2

$$\begin{array}{r} \text{china} \\ 1272.3 \\ - 1221.6 \\ \hline 50.7 \end{array} \quad \begin{array}{r} \text{US} \\ 279.5 \\ - 268.0 \\ \hline 11.5 \end{array}$$

- a. Jennifer can be right because China's population increased by more individual people than the U.S. did. Katy can be correct because the U.S. rate of increase is higher than China's.

$$1221.6x = \frac{1272.3}{1221.6} \approx 1.0415$$

China

$$268.0x = \frac{279.5}{268} \approx 1.0429$$

United States

b.

$$268.0 + \frac{279.5}{268}x = 300 \quad (x-1)$$

$$82 = \frac{268}{279.5} = \underline{30.683} \approx 31$$

$$\frac{20.683}{5} = 6$$

After about ~~5~~<sup>6</sup> year period the United States' Population will reach 300 million.

41.

2

Well China increased more in numbers of population. The United States increased more percent wise.

China's difference 50.7  
 US " " 11.5

percent (U.S)

$$279.5 - 268.0 = \frac{11.5}{268.0} = 4.3\%$$

percent (China)

$$\frac{50.7}{1221.6} = 4.2\%$$

41.

1

A- They both can't be correct. Jennifer is right, China is increasing by 10.14 million people a year, which was a total of 50.7 million people between 1997 and 2002. While the U.S. is only increasing by 2.5 million per year which is only a total of 11.5 million between 1997 and 2002.

B- Increase by 11.5 each 5 year period.

$$279.5 + \underline{11.5} + \underline{11.5} = 302.5$$

after 2 more 5 year periods

41.

a) Jennifer can be correct because the actual rate at which the population is growing is faster. Katy could be correct because more people could be coming to the United States increasing its population.

b) 2 more 5-year time periods would be needed for the United States to reach 300 million.